

How the 5 Counting Principles lay the foundation for flexible thinking in later grades.

Flexible thinking - thinking differently about the same thing - is just as important with very young children as it is in higher level concepts. One area where children can begin to think flexibly is counting. Given opportunities to discover that they can count different objects as one group and that counting objects in a different order doesn't make a difference, will help students realize there is more than one way of thinking in math.

Teaching students to count is more complicated than it sounds. It is more than just rote counting or calling numbers aloud in sequence. In 1978, researchers Rochel Gelman and Randy Gallistel introduced the idea of five separate and distinct principles that children need to understand in order to be proficient counters. (see the list below) Children often learn to count without any direct instruction. However, many children do not learn the complexities of counting if they are not introduced to the ideas.

If we are purposeful and explicit in our teaching, all children can learn to fully understand counting through play. By recognizing what they know, and determining what they need to know next, we can provide purposeful activities to strengthen and move their learning forward. We can do this by promoting curiosity through our conversations and encouraging a dialogue of discovery among students.

The 5 Counting Principles are often invisible unless you know what to look for and what to ask. The following pages describe these ideas so that you may observe your students and encourage their growth.

The Counting Principles

"How to Count"

1. The 1:1 Counting Principle
2. The Stable-Order Principle
3. The Cardinality Principle

"What to Count"

4. The Abstraction Principle
5. The Order-Irrelevance Principle

Numeral Ranges

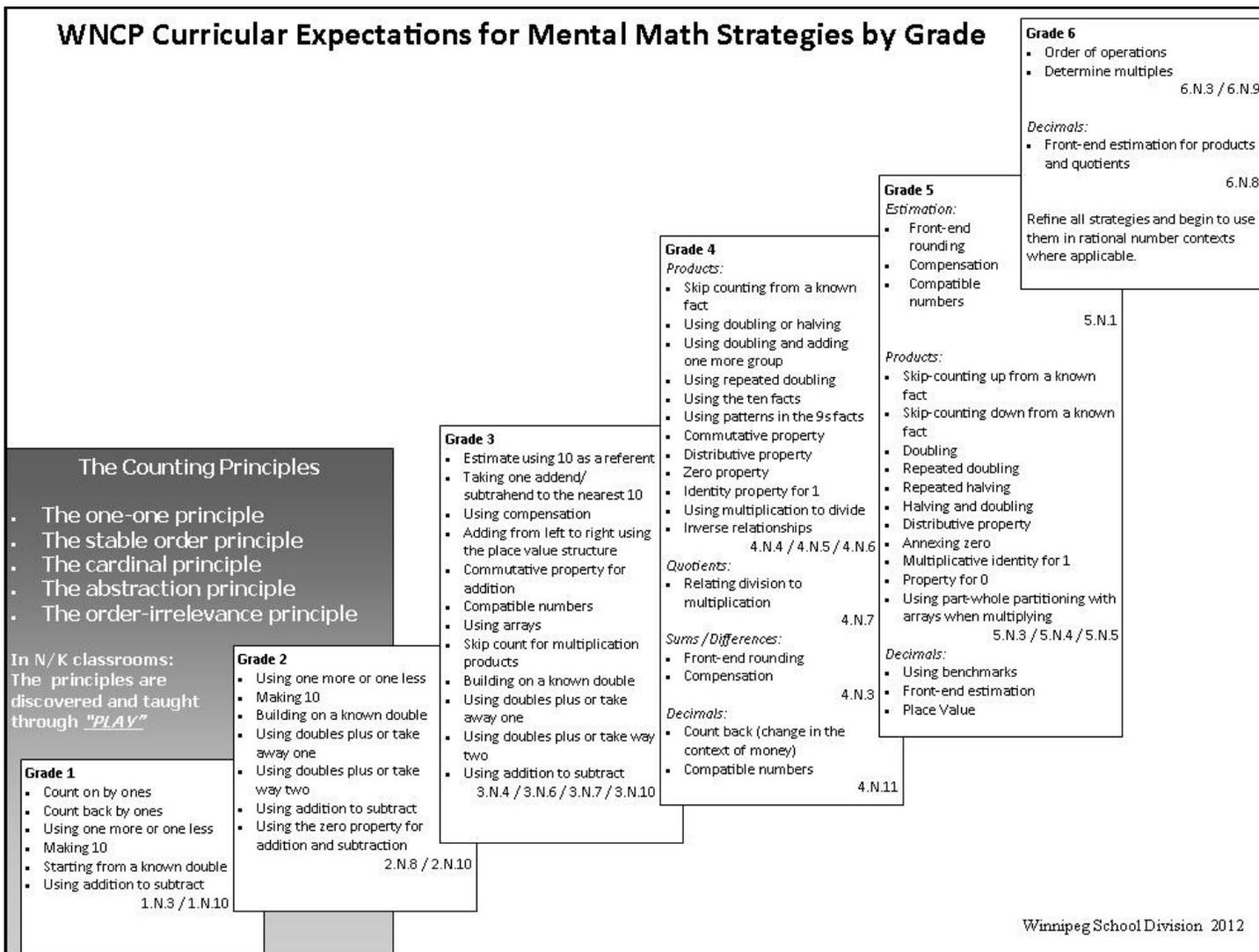
Nursery:

- Students should have a strong understanding - both forward and backward - to **5**, before they move on to **10**.

Kindergarten:

- Students should have a strong understanding - both forward and backward - to **10**, before they move on to **20**.

WNCP Curricular Expectations for Mental Math Strategies by Grade



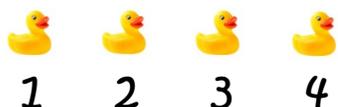
The above image shows how the 5 Counting Principles support the development of Mental Math Strategies. Before children can learn to “count on,” they need be able to confidently count forward. Before children can learn to subtract, they need be able to confidently count backward.

Researchers point out that it is important for children to fully explore small numeral ranges (to 5), before they move on to work within a higher range (to 10). That means that students need to count, put together (add), take apart (subtract), and compare sets to 5. After they are confident to 5, then they can move on to do the same for sets to 10, then to 20.

Developing a strong foundation of counting, then creating picture representations of a set, and then visualizing a set in their minds, are all important steps before children use **only** numbers. Understanding that two smaller sets can make one larger set (part-part-whole) is also a very important concept for students. As well, they need to understand the opposite--that a whole can be made up of two smaller sets. For a strong foundation to develop, students need repeated practice in different settings and encouragement to talk about their ideas.

The 1 to 1 Principle

Description: You say one number, and only one number, for each object.



Prior Knowledge:

- The words to orally rote count.

Key Points:

- Modeling with individual or small groups of students by counting items during play.
- Modeling in larger groups and encouraging students to show fingers or perform actions while reading counting books or counting rhymes.

Strategies that Support Student Learning:

- Moving objects when counting.
- Matching items with pictures.
- When playing an instrument such as rhythm sticks, a drum, or a xylophone, students can record one mark on a paper for every sound.
- Helping to set the table for snack or in the kitchen play centre.
- Counting the number of words someone says or the number of questions the class asks.
- Counting letters in someone's name.

When observing students during play...

...if you see:

- A child playing with a bus putting animals or people inside.
- Two children playing with a ball.

...then ask:

- How many animals can you fit inside the bus?
- I wonder if you could pass the ball back and forth 20 times?

Consider this: Children need to experience counting in a variety of ways to determine quantity. If we consistently ask, "How many are there?" they might interpret that to mean, "Please count." Varying the questions we ask, helps children see that one way of determining quantity is by counting. They could determine quantity by subitizing or relating the items to another set.

The Stable-Order Principle

Description: There is a set of counting words that everyone uses and never changes.

one, two, three, four, five...

Prior Knowledge:

- The words to orally rote count.

Key Points:

- Some feel we introduce the symbol for a quantity before children are ready. Helping students have a firm grasp on the quantity associated with each number before we formally introduce the symbolic form, may strengthen their number sense.

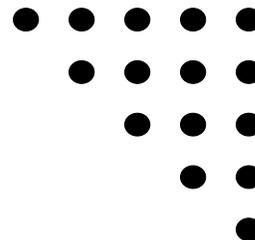
For example:

“three” is not “3” but rather

“three” is 

Strategies that Support Student Learning:

- Putting pictures of items in order from smallest to largest, based on quantity, and counting them forwards and backwards.
- Organizing objects in order, without numbers at first, then adding the symbols later.



When observing students during play...

...if you see:

- A child miscounting either orally by rote or with objects.

...then ask:

- I'm going to make a mistake when I'm counting. Can you tell me what number I missed?

Consider this: There are some numbers that do not represent a quantity and might be confusing to children. Some are keeping track (hours in a day—clock; days in a month—calendar) and some have no connection to quantity at all (telephone numbers, house numbers). Helping students see the difference will strengthen the attachment of quantity to the symbolic form of numbers.

The Cardinal Principle

Description: The last number spoken, when counting a set of items, tell how many items are in the set.

1, 2, 3, 4, 5, ⑥

Prior Knowledge:

- The words to orally rote count.
- The 1:1 Principle
- The Stable-Order Principle

Key Points:

- Check that students begin counting at “1” and give repeated examples of how the counting procedure does not work if you start at another number.
- Cardinality is sometimes difficult to assess. One tried and true way is to ask students to determine the quantity of a set and then ask them to put the same amount into your hands - if they have to recount, then they do not have a firm understanding of Cardinality.
- Subitizing occurs before counting and is another way of reinforcing cardinality.

Strategies that Support Student Learning:

- Creating a set of items to match a number. (This will also help with learning to compare quantities using the words the same, more, fewer.)
- Show Me - The 1st child says, “Show me a set of 4.” The 2nd child builds a set to 5 and the 1st child checks for accuracy. Then, it is the 2nd child’s turn. He/she says, “She me a set of 5.” This can go on back and forth with the children choosing a different number each time.

When observing students during play...

...if you see:

- A child putting cars in the carwash.

...then ask:

- Can you put the same amount of cars in the garage?

Consider this: Student ability to count and determine cardinality is within different numeral ranges at different times. For example, they can count from 1—10 accurately, but not to 20. It is the same with Cardinality—just because they can determine how many are in a set to 5 does not mean that they can do it to 10.

The Abstraction Principle

Description: It does not matter *what* you count, the *way* we count is always the same.



Prior Knowledge:

- The words to orally rote count.
- The 1:1 Principle
- The Stable-Order Principle
- The Cardinal Principle (A child might learn these two Principles at the same time.)

Key Points:

- Sometimes when presented with different objects, children state that larger items have more value. Consider asking students to count stuffed animals that are very different in size, such as an elephant, a mouse, and a bird. Demonstrate that the stuffed animals in the set all have the same value of “1.”
- Temporal Patterns - listening to pennies drop in a jar and counting, or listening to drumbeats and counting, helps the development of this principle.

Strategies that Support Student Learning:

- Counting non-tangible things such as:
 - sounds
 - actions
 - words people say
 - questions people ask
 - steps people take
- Matching sets of different items with the same quantity.

When observing students during play...

...if you see:

- A child playing with a collection of items that are not the same.

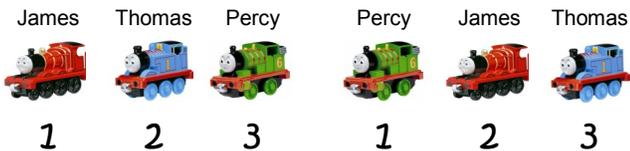
...then ask:

- I wonder if we could count these altogether?
- If I have 5 trucks and you have 5 blocks do we have the same amount of toys?

Consider this: Invite students to communicate or explain their thinking by regularly asking them “How do you know?” You will gain a greater understanding of their strategies and ability.

The Order-Irrelevance Principle

Description: It does not matter in which order you count, the number in the set does not change.



Prior Knowledge:

- The words to orally rote count.
- The 1:1 Principle
- The Stable-Order Principle
- The Cardinal Principle
- The Abstraction Principle (A child might learn these two Principles at the same time.)

Key Points:

- **Researchers have found that this is the most under-learned of all five principles.** Students may have the misconception that the order **does** matter until as late as Grade 4.
- Some children can know this principle, but not be strong in others like Cardinal and Abstraction. Working through the ideas behind the principles might strengthen all three at the same time.

Strategies that Support Student Learning:

- Counting sets of items from
 - left-to-right
 - right-to-left
 - top-to-bottom
 - bottom-to-top
- Counting sets of different coloured items or different objects in a different order.

When observing students during play...

...if you see:

- A child counting a set of items.

...then ask:

- Can you predict how many there will be if you count these items starting at this end?

Consider this: Researchers believe that there is a strong connection between this principle and Part-Whole Thinking. By understanding that the order size, shape, or colour does not matter, students will be more secure in the fact that the order of the numbers (symbolic parts) does not matter. For example: 2 and 3 is 5 **and** 4 and 1 is 5 **and** 3 and 2 is 5 **and** 1 and 4 is 5.

Resources

Winnipeg School Division

- Mathematics Website
- Background Information for the Learning Pathway; Global Stage: Counting as a Strategy
- Bibliography - Mathematics Assessment Binder

Books

- *Growing Mathematical Ideas in Kindergarten* by Linda Schulman Dacey and Rebeka Eston
- *The Young Child and Mathematics* by Juanita Copley
- Big Ideas K-3 by Marian Small

Websites

- Subitizing: What is it? Why teach it?
<http://gse.buffalo.edu/fas/clements/files/Subitizing.pdf>
- The principal counting principles
<https://www.ncetm.org.uk/public/files/712850/The+principal+counting+principles.pdf>
- Counting on Counting
www.nzmaths.co.nz/resource/counting-counting

Videos

- Counting is Complicated Part 1
<http://www.youtube.com/watch?v=rsKNrnIfXt4>
- Counting is Complicated Part 2
<http://www.youtube.com/watch?v=xssBJpOBecs&feature=relmfu>
- Counting is Complicated Part 3
http://www.youtube.com/watch?v=WVfwBQe_IJE&feature=relmfu
- The Math in Blocks
<http://www.youtube.com/watch?v=gsDY6qftzQk&feature=relmfu>
- Learning math early -- it's big!
<http://www.youtube.com/watch?v=1ljesoJJTp0&feature=relmfu>